

Compensation for large tensor modes with iso-curvature perturbations in CMB anisotropies

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Abstract

Recently, BICEP2 has reported the large tensor-to-scalar ratio $r = 0.2^{+0.07}_{-0.05}$ from the observation of the cosmic microwave background (CMB) B-mode at degree-scales. Since tensor modes induce not only CMB B-mode but also the temperature fluctuations on large scales, to realize the consistent temperature fluctuations with the Planck result we should consider suppression of scalar perturbations on corresponding large scales. To realize such a suppression, we consider anti-correlated iso-curvature perturbations which could be realized in the simple curvaton model.

1 Introduction

Recently, BICEP2 collaboration has reported the detection of the cosmic microwave background (CMB) B-mode polarization at the degree scales at 7.0σ [1, 2]. The detected degree scale B-mode polarization is consistent with the primordial gravitational waves origin and the corresponding value of the tensor-to-scalar ratio, which represents the amplitude of the primordial gravitational waves, is $r = 0.2^{+0.07}_{-0.05}$. The amplitude of the primordial gravitational wave is a direct probe of the energy scale of the inflation, which is given by

$$H_{\text{inf}} \simeq 1.22 \times 10^{14} \text{ GeV} \left(\frac{r}{0.2} \right)^{1/2}. \quad (1)$$

Based on this result, there have appeared a number of interesting papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

However, there is a tension between BICEP2 and Planck results. The latter gives the upper bound on the tensor-to-scalar ratio as $r < 0.11$ obtained from the combined analysis of the CMB temperature fluctuations and other data sets [20, 21]. The key point to resolve this discrepancy is a suppression of the scalar perturbations on large scales enough to compensate for the contribution from the large tensor modes. As an example, running spectral index is discussed in BICEP2 paper and it is necessary to consider relatively large (in terms of absolute value) negative running such as $dn_s/d\ln k \sim -0.02$ to resolve the tension, which is much larger than the expected value in the standard slow-roll inflation model. In Ref. [14], the authors consider the anti-correlation between the scalar and tensor perturbations instead of the running spectral index.

In this paper, we alternatively consider the anti-correlated iso-curvature perturbation to compensate for the contribution of the large tensor modes [22]. Such iso-curvature perturbations could be realized in the simple curvaton model which has both cold dark matter (CDM) and baryon isocurvature perturbations. This paper is organized as follows. In the next section, we show our basic idea of compensation for the tensor contribution with the iso-curvature perturbations. In section 3, in the context of the curvaton scenario we present a concrete model where the required iso-curvature perturbation could be realized, and we devote the final section to summary.

2 Compensation caused by iso-curvature perturbations

In this section let us describe the basic idea. We focus on the compensation for the large tensor modes due to the iso-curvature perturbations. Owing to the damping behavior of the tensor and iso-curvature perturbations, both perturbations give similar contributions to large scale temperature fluctuations. On large scales, the temperature anisotropy induced from scalar perturbation is mainly sourced from the Sachs-Wolfe effect as

$$\left(\frac{\Delta T}{T}\right)_{\text{sw}} = -\frac{1}{5}\zeta - \frac{2}{5}\mathcal{S}_m, \quad (2)$$

where ζ is the adiabatic curvature perturbation on uniform energy density slice during the radiation-dominated era, and \mathcal{S}_m is the total matter iso-curvature perturbations. The matter component consists of the CDM and baryons and hence we have

$$\mathcal{S}_m = \frac{\Omega_{\text{CDM}}}{\Omega_m}\mathcal{S}_{\text{CDM}} + \frac{\Omega_b}{\Omega_m}\mathcal{S}_b, \quad (3)$$

where \mathcal{S}_{CDM} and \mathcal{S}_b are respectively CDM and baryon iso-curvature perturbations, and Ω_i ($i = \text{CDM}, b, \text{ and } m$) with $\Omega_m = \Omega_{\text{CDM}} + \Omega_b$ is a density parameter of each component. As we have mentioned, the tensor perturbations also contribute the CMB temperature fluctuations. On large scales, the tensor-contribution can be approximately written as [14]

$$\left(\frac{\Delta T}{T}\right)_{\text{tens}} \simeq \frac{1}{2}h_{ij}n^in^j, \quad (4)$$

where n is a unit vector. Following Ref. [14], including all contributions to the temperature anisotropies we obtain

$$\begin{aligned} \left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle &\propto \mathcal{P}_\zeta \left(1 + 4\frac{\mathcal{P}_{\mathcal{S}_m}}{\mathcal{P}_\zeta} + 4\frac{\mathcal{P}_{\zeta\mathcal{S}_m}}{\mathcal{P}_\zeta} + \frac{5}{6}\frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \right) \\ &= \mathcal{P}_\zeta \left(1 + 4B_m^2 + 4B_m \cos \theta_m + \frac{1}{6}\left(\frac{r}{0.2}\right) \right), \end{aligned} \quad (5)$$

where \mathcal{P}_i ($i = \zeta, \mathcal{S}_m, T$) is the power spectrum of each component, and $\mathcal{P}_{\zeta\mathcal{S}_m}$ is a cross-power spectrum of the adiabatic and iso-curvature perturbations. Here, we also introduce parameters related with the iso-curvature perturbations as

$$B_m \equiv \sqrt{\frac{\mathcal{P}_{\mathcal{S}_m}}{\mathcal{P}_\zeta}}, \quad \cos \theta_m \equiv \frac{\mathcal{P}_{\zeta\mathcal{S}_m}}{\sqrt{\mathcal{P}_\zeta\mathcal{P}_{\mathcal{S}_m}}}. \quad (6)$$

From Eq. (5), to compensate for the tensor contribution with the iso-curvature perturbation, the following relation is required:

$$4B_m^2 + 4B_m \cos \theta_m + \frac{1}{6} \left(\frac{r}{0.2} \right) = 0. \quad (7)$$

To realize this requirement, we at least need anti-correlated iso-curvature perturbations, that is, $\cos \theta_m < 0$, and with $\cos \theta_m = -1$ we obtain

$$B_m = \frac{1}{2} \left(1 \pm \sqrt{1 - \frac{1}{6} \left(\frac{r}{0.2} \right)} \right) > 0. \quad (8)$$

If we chose the BICEP2 result, $r = 0.2$, we have $B_m \simeq 0.04$ or 0.96 . In Ref. [22], by using the WMAP3 and other data sets the constraints on the value of B_m and r are respectively given by $B_m \cos \theta_m \geq -0.13$ and $r \leq 0.49$. Hence, our naive estimation seems to be agreement with these constraints for $B_m = 0.04$.

3 How to realize required anti-correlated iso-curvature perturbations?

Let us consider the concrete example which can realize the required anti-correlated iso-curvature perturbations as we have shown in the previous section. Here, we consider the curvaton scenario [23, 24, 25] as the mechanism of generating the iso-curvature perturbations [26, 27, 28, 29, 30, 31, 32].

In the curvaton scenario, the adiabatic curvature perturbations could be induced not only from the inflation fluctuations but also fluctuations of a light scalar field other than inflaton called as curvaton. The curvaton field which is subdominant component of the Universe has acquired the fluctuations during inflation, and then starts to oscillate when the Hubble parameter becomes equal to the mass of the field. Such oscillating scalar field behaves as non-relativistic matter. Hence, during radiation dominated era, the energy density of the curvaton becomes larger compared with the total energy density of the Universe and simultaneously the adiabatic fluctuations also evolves due to the iso-curvature perturbations sourced from the curvaton fluctuations. Finally, after the curvaton decays the adiabatic curvature perturbations stay constant in time. The total adiabatic curvature perturbations after the curvaton decay is given by

$$\zeta = \zeta_{\text{inf}} + \frac{f_{\text{dec}}}{3} \mathcal{S}_\sigma, \quad (9)$$

where $f_{\text{dec}} (= 3\rho_\sigma / (3\rho_\sigma + 4\rho_r)|_{\text{decay}})$ is a parameter related with the ratio between the curvaton and the radiation energy density at the curvaton decay, ζ_{inf} is the curvature perturbations induced from the inflaton fluctuations and \mathcal{S}_σ is the curvaton iso-curvature perturbations which is given by $\mathcal{S}_\sigma = 3(\zeta_\sigma - \zeta_{\text{inf}})$ with ζ_σ being the curvature perturbations on the uniform energy density slicing of the curvaton.

As for the residual iso-curvature perturbations, \mathcal{S}_{CDM} or \mathcal{S}_b , it depends on the generation process of the CDM and baryons. In the case where the CDM/baryons have been produced from the inflaton decay, the curvature perturbation on the uniform CDM/baryons energy density slice, $\zeta_{\text{CDM/b}}$, should be equal to ζ_{inf} and then the residual iso-curvature perturbations defined as $\mathcal{S}_{\text{CDM/b}} \equiv 3(\zeta_{\text{CDM/b}} - \zeta)$ are given by

$$\mathcal{S}_{\text{CDM/b}} = -f_{\text{dec}}\mathcal{S}_\sigma. \quad (10)$$

From this expression, we find that this case has the possibility of generating required anti-correlated iso-curvature perturbations. On the other hand, in the case where the CDM/baryons have been produced from the curvaton decay, we have $\zeta_{\text{CDM/b}} = \zeta_\sigma$ and then the residual iso-curvature perturbations are given by

$$\mathcal{S}_{\text{CDM/b}} = (1 - f_{\text{dec}})\mathcal{S}_\sigma. \quad (11)$$

In this case, it is difficult to realize the anti-correlated iso-curvature perturbations because of the fact that $f_{\text{dec}} < 1$.

Let us focus on the former case and assume that both of CDM and baryons have been produced from the inflaton decay. In this case, the parameters introduced in the previous section are respectively written as

$$B_m = \sqrt{\frac{f_{\text{dec}}^2 R}{\left(1 + \frac{f_{\text{dec}}^2}{9}R\right)}}, \quad \cos \theta_m = -\frac{f_{\text{dec}}^2 R}{3\sqrt{\left(1 + \frac{f_{\text{dec}}^2}{9}R\right)} f_{\text{dec}}^2 R}, \quad (12)$$

where R represents the ratio of the amplitude of \mathcal{S}_σ to that of ζ_{inf} defined as $R \equiv \mathcal{P}_{\mathcal{S}_\sigma} / \mathcal{P}_{\zeta_{\text{inf}}}$. Substituting the above expressions into the left hand side of Eq. (7), we have

$$4B_m^2 + 4B_m \cos \theta_m + \frac{1}{6} \left(\frac{r}{0.2} \right) = \frac{8}{3} \frac{f_{\text{dec}}^2 R}{1 + \frac{f_{\text{dec}}^2}{9}R} + \frac{1}{6} \left(\frac{r}{0.2} \right). \quad (13)$$

Since all parameters are positive definite, this case can not realize the required iso-curvature perturbations. As we have mentioned, in another case where both of CDM and

baryons have been produced from the curvaton decay the expected residual iso-curvature perturbations cannot be the required one.

Next, let us consider the mixed scenario where the CDM and baryons are sourced from the inflaton and curvaton decay, respectively. In such case, we have

$$\mathcal{S}_{\text{CDM}} = -f_{\text{dec}}\mathcal{S}_\sigma, \quad \mathcal{S}_\text{b} = (1 - f_{\text{dec}})\mathcal{S}_\sigma, \quad (14)$$

and the total matter iso-curvature perturbations are given by

$$\mathcal{S}_\text{m} = \left[-f_{\text{dec}} \frac{\Omega_{\text{CDM}}}{\Omega_\text{m}} + (1 - f_{\text{dec}}) \left(1 - \frac{\Omega_{\text{CDM}}}{\Omega_\text{m}} \right) \right] \mathcal{S}_\sigma. \quad (15)$$

Taking the parameter f_{dec} to be equal to $1 - \Omega_{\text{CDM}}/\Omega_\text{m}$, then the total matter iso-curvature perturbations is completely cancelled even as each component has the iso-curvature perturbations, so-called compensated iso-curvature perturbations [33, 34, 35, 36, 37]. On the other hand, to realize the compensation for the large tensor mode with the iso-curvature perturbation we do not need completely compensated iso-curvature perturbations but the partially compensated ones. By using the above expression Eq. (15), we have

$$B_\text{m} = \sqrt{\frac{\left(1 - \frac{\Omega_{\text{CDM}}}{\Omega_\text{m}} - f_{\text{dec}}\right)^2 R}{\left(1 + \frac{f_{\text{dec}}^2}{9} R\right)}}, \quad \cos \theta_\text{m} = \frac{f_{\text{dec}} \left(1 - \frac{\Omega_{\text{CDM}}}{\Omega_\text{m}} - f_{\text{dec}}\right) R}{3\sqrt{\left(1 + \frac{f_{\text{dec}}^2}{9} R\right) \left(1 - \frac{\Omega_{\text{CDM}}}{\Omega_\text{m}} - f_{\text{dec}}\right)^2 R}}. \quad (16)$$

Substituting the above expression into the left hand side of Eq. (7), we have

$$4 \frac{\left(1 - \frac{\Omega_{\text{CDM}}}{\Omega_\text{m}} - f_{\text{dec}}\right) R}{1 + \frac{f_{\text{dec}}^2}{9} R} \left(1 - \frac{\Omega_{\text{CDM}}}{\Omega_\text{m}} - \frac{2}{3} f_{\text{dec}}\right) + \frac{1}{6} \left(\frac{r}{0.2}\right) = 0. \quad (17)$$

As shown in Fig. 1, we can easily find that the above equation has solutions for large R case. In the limiting case where the curvaton contribution dominates over the adiabatic perturbations, that is, $f_{\text{dec}}^2 R/9 \gg 1$, for the Planck best-fit values of Ω_{CDM} and Ω_m and the BICEP2 value of r , that is, $\Omega_{\text{CDM}}/\Omega_\text{m} = 0.845$ and $r = 0.2$, we find that $f_{\text{dec}} \simeq 0.16$ and 0.23 are solutions for Eq. (17). Hence, in such case we can realize the required iso-curvature perturbations to compensate for the tensor contribution to the temperature anisotropies, which makes the BICEP2 result consistent with the Planck result. As is well-known, although the parameter f_{dec} is an important parameter to describe the primordial non-Gaussianity and it is strongly constrained as $f_{\text{dec}} > 0.15$ [38], this constraint is consistent with the required values $f_{\text{dec}} \simeq 0.16$ and 0.23 .

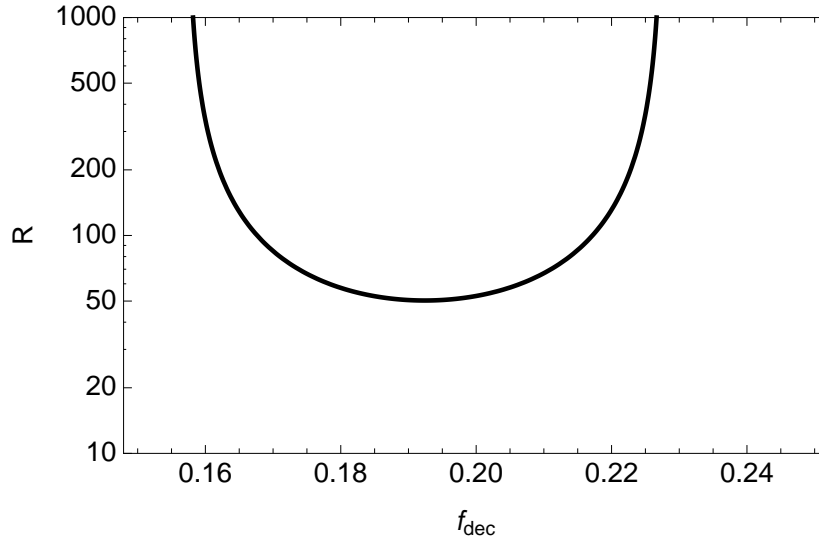


Figure 1: The solution for Eq. (17) in $f_{\text{dec}}-R$ plane.

We would like to note that the BICEP2 result basically indicates that even if we consider the curvaton scenario we hardly neglect the inflaton contribution due to the large tensor-to-scalar ratio [11]. As shown in Fig. 1, for minimum R , we can also have a solution at $R \simeq 50$ and $f_{\text{dec}} \simeq 0.19$. For these parameters, the ratio R_{curv} of the power spectra of the curvature perturbations from the inflaton to the curvaton is given by $R_{\text{curv}} = f_{\text{dec}}^2 R/9 \simeq 0.2$ [see, Eq. (9)] and hence we can realize the required iso-curvature fluctuations in case where the curvaton contributions are only 20% of the inflaton contributions.¹

We find that the required iso-curvature perturbations can be also realized for the opposite case, that is, the case where CDM and baryons are sourced from the curvaton and inflaton decay, respectively. In such case, the solution can be obtained for relatively large f_{dec} and small R . For example, we have a solution for $f_{\text{dec}} \simeq 0.95$ and $R \simeq 2$, and this means $R_{\text{curve}} \simeq 0.2$. Hence, in this case we can also realize the required situation by considering only 20% curvaton contributions.

¹In case $R_{\text{curve}} \gtrsim 1$ the slow-roll parameter $\epsilon = \dot{H}_{\text{inf}}/H_{\text{inf}}^2$ (H_{inf} :Hubble during inflation) should be large to suppress the inflaton contribution, which leads to red-tilted spectral index of the curvaton perturbations.

4 Summary

Recent BICEP2 report about the detection of the degree-scale CMB B-mode polarization has revealed the energy scale of the inflation and opened a window into the deep understanding of the physics of the early Universe.

However there still exist a tension between the BICEP2 result $r = 0.2^{+0.07}_{-0.05}$ and Planck result $r < 0.1$ from the observations of the CMB temperature fluctuations. In order to resolve this discrepancy, we need to consider the suppression of the large scale scalar perturbations. In this paper, instead of considering the large negative running discussed in the BICEP2 paper, we consider the anti-correlated iso-curvature perturbations. We present a concrete example to realize the required anti-correlated iso-curvature perturbations in the context of the curvaton scenario. We just show the possibility by using the rough estimate and hence it should be interesting to perform more detailed analysis by using e.g., Markov Chain Monte Carlo methods. We leave such detailed analysis as a future issue.

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